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TIME-FREQUENCY CHARACTERIZATION OF ATRIAL ARRHYTHMIAS USING PRINCIPAL DECOMPOSITION

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Abstract— **A new noninvasive technique for atrial arrhythmia analysis is presented which, based on time–frequency analysis and principal decomposition, produces trends of the atrial signal characteristics using the surface ECG. New homogeneity measures are introduced in order to continuously measure how well the decomposed functions represent the atrial signals. A database containing signals from 20 patients with different atrial arrhythmias (mainly atrial fibrillation) was analyzed. It was found that the method is very well-suited for characterizing the signals and it is expected that the resulting functions may be useful for separating different arrhythmias.**

I. Introduction

It is desirable to find non-invasive methods for characterization and classification of atrial arrhythmias, including tachycardia, flutter and fibrillation. Information contained in the atrial activity must, in some suitable way, be quantified to accomplish this task. So far, the main efforts in this field have been directed towards atrial fibrillation analysis although the same methods in many cases can be used for flutter and tachycardia. In the atrial fibrillation case, the atrial activity in the ECG have conventionally been classified into "coarse/medium/fine" fibrillation, see e.g. [1], [2], to provide a general description of structure and amplitude. The repetition rate (or atrial cycle length) of the f–waves in the ECG has also been investigated and serves as an index of the degree of atrial organization [3], [4]. Estimation of the average repetition rate can be based on spectral analysis. Such an approach gives a general picture of the signal by providing information about the average repetition rate by means of the peak location, the variation in the rate by the width of the peak and the average signal energy by the peak amplitude. This method is simple but provides valuable clinical information.

Recently, we suggested time-frequency analysis (TFA) for a more detailed temporal characterization of variations in the repetition rate, [5]. The potential of having a high temporal resolution was illustrated by two situations: a few patients were found for which the typical irregular rhythm of atrial fibrillation was suddenly interrupted by short intervals of a different, more regular rhythm. Another situation is that changes in repetition rate can be investigated during different interventions. The strategy that was chosen in order to analyze the signals on a second-to-second basis was to use an iterative cross-Wigner-Ville distribution (XWVD). In some patients, the XWVD revealed very large variations in repetition rate (e.g., varying from 5 to 7 Hz) whereas very small variations were observed in others.

However, the XWVD models the frequency variations as a frequency-modulated sinusoid which has a low-pass effect on the trends. Further, it only uses the energy in the fundamental frequency and is therefore not capable of tracking the shape of the signals as described by its harmonics. Another limitation is that the computational complexity is relatively high.

Atrial signals may be nonstationary but are repetitive and thus they can during short intervals be represented by a fundamental which reflects the repetition rate and a harmonic pattern which reflects the shape of the fibrillatory waveform. One way to achieve a detailed feature extraction in the time-frequency plane for this type of signals is to decompose the time-frequency distribution into descriptive functions ("principal components"): spectral profile, frequency shift trend and amplitude scaling trend, [6].

This paper addresses the problem of characterizing different atrial arrhythmias using a modified TFA-based principal component decomposition of residual ECGs in order to extract clinically interesting features such that different arrhythmias or different intervals during one arrhythmia can be differentiated. The goal is to find a set of trends that can serve as a basis for classification of atrial arrhythmias.

II. Methods

We propose a TFD-based characterization of residual ECGs. The methods consists of four parts: a preprocessing stage which performs prefiltering and QRST cancellation in order to produce an atrial signal. In the second stage, the time-frequency distribution is computed and decomposed into a set of descriptive functions. Finally, a postprocessing stage is used to analyze the spectral content of the frequency trend.

A. Preprocessing

A spatiotemporal QRST cancellation scheme was used [7]. Since the spectral content of interest in the resulting residual ECG signal is well below 25 Hz, the residual ECG can be downsampled from 1 kHz to 50 Hz. This operation considerably reduces the amount of data to be processed. The main parts of the ventricular activity (QRS complex and T wave) is removed in the residual ECG signal and thus this signal contains primarily atrial activity i.e. P waves during sinus rhythm and f-waves during atrial fibrillation.

B. Time-frequency analysis

In the present paper, we focus on atrial tachycardia, atrial flutter and atrial fibrillation and for this purpose only the frequency interval between 2.5-25 Hz is of interest. This interval is chosen such that slow tachycardias down to 2.5

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Abstract

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Hz can be represented. All these types of atrial signals have in common that they are repetitive. During tachycardia the cycles are often of equal length. For fibrillation, the cycle length may vary very rapidly and the signal is referred to as being nonstationary. However, for short intervals (typically 1-2 seconds) atrial fibrillation signals can be viewed as approximately stationary. The result is that when analyzing the spectral content of all these types of different atrial signals on a second-to-second basis the result will be a fundamental and a number of harmonics reflecting the shape of the waveforms. The harmonic pattern have negligible energy above 25 Hz. Another reason to use the lower limit at 2.5 Hz is that possible small QRST residuals appear at low frequencies depending on the RR interval and the window length. If the QRST residuals are of high amplitude or occur in a periodic way they may have spectral content above 2.5 Hz.

Let the frequency vector $\mathbf{f} = [f_0 \dots f_{N_f-1}]^T$ denote an increasing sequence (length N_f) of normalized frequencies. The corresponding DFT matrix, $\mathbf{F}$, (N_f-by-N_w where N_w is the signal window length) can then be written

$$\mathbf{F} = \left[\begin{array}{cccc} \mathbf{1} & e^{-j2\pi\mathbf{f}} & e^{-j2\pi\mathbf{f}2} & \dots & e^{-j2\pi\mathbf{f}(N_w-1)} \end{array} \right] \quad (1)$$

where $\mathbf{1}$ is a column vector of length N_f. If the frequencies are uniformly distributed the DFT matrix is unitary. In the present method, the Short-Term Fourier transform (STFT) with a logarithmic frequency scale for the frequency interval 2.5-25 Hz such that a doubling in frequency for all frequencies corresponds to the same number of frequency bins. The motivation for this is to be able to compare the harmonic pattern for intervals with different fundamental frequency (or atrial cycle length). A signal window length of $N_w = 128$, i.e, about 2.5 seconds, is used. The frequency vector was defined as

$$\mathbf{f} = \left[\begin{array}{cccc} 2.5 & 2.5 \cdot 10^{\frac{1}{N_f}} & \dots & 2.5 \cdot 10^{\frac{N_f-1}{N_f}} \end{array} \right] \quad (2)$$

where $N_f = N_w$. Further, the diagonal entries in the diagonal matrix $\mathbf{H}$ (N_w-by-N_w) represent a window function (here chosen as Hamming). The observed signal, $x(n)$, is represented by a data matrix, $\mathbf{X}$, in which the columns are overlapping data sequences. For a window distance L and a total signal length of K window intervals (total signal length is thus $(K-1) - by - L + N_w$) the data matrix (N_w-by-K) is given by

$$\mathbf{X} = \left[\begin{array}{ccc} x(0) & \dots & x((K-1)L) \\ \vdots & \vdots & \vdots \\ x(N_w-1) & \dots & x((K-1)L+N_w-1) \end{array} \right] \quad (3)$$

where column k represents the signal in the k:th window interval.

The STFT of the observed signal $x(n)$ can be written as

$$\mathbf{Q} = \mathbf{FHX} \quad (4)$$

where $\mathbf{Q}$ is N_f-by-K and the k:th column contains the spectrum, $\mathbf{q}_k$, for the k:th window interval.

$$\mathbf{Q} = [\mathbf{q}_0 \quad \dots \quad \mathbf{q}_{N-1}] \quad (5)$$

C. Principal decomposition

A time-frequency distribution, $\mathbf{Q}$, of a nonstationary but cyclic signal can, using principal decomposition, be divided into three trends: a *spectral profile function*, ϕ, a *frequency-shift function*, θ_k, and an *amplitude function*, a_k, using an iteration method, which shifts the spectra for the different window intervals such that the first principal component of the correlation matrix based on all spectra represents as much of the energy as possible, [6]. This is achieved when the peaks for all spectra are at the same location. The shift needed for each of the spectra is the frequency-shift function, θ_k. The inner product between the first principal component and each spectrum, shifted such that the peaks match, reflects the amplitude, a_k, of the signal in that window interval. The iterative procedure is initialized with, $\mathbf{Q}^{(0)} = \mathbf{Q}, i = 0$. The correlation matrix, $\mathbf{R}^{(i)}$, for the distribution , $\mathbf{Q}^{(i)}$, is given by

$$\mathbf{R}^{(i)} = \mathbf{Q}^{(i)}\mathbf{Q}^{(i)H} \quad (6)$$

The eigenvector corresponding to the largest eigenvalue, $\phi^{(i)}$, can be calculated using the iterative "power method". An extended eigenvector, $\tilde{\phi}^{(i)}$, with Θ (maximum frequency shift) extra samples in both the beginning and the end is created in order to allow selection of different parts of the vector. These extra samples are set to the same value as the start and end samples of $\phi^{(i)}$ respectively. There are now $2\Theta + 1$ possible sets of N_f samples that can be selected from $\tilde{\phi}^{(i)}$. The selection is done using a frequency shift matrix $\mathbf{J}_\theta$ defined as

$$\mathbf{J}_\theta = [\mathbf{0}_{N_f \times (\Theta+\theta)} \quad \mathbf{I}_{N_f \times N_f} \quad \mathbf{0}_{N_f \times (\Theta-\theta)}] \quad (7)$$

which selects N_f samples from $N_f + 2\Theta$.

Maximization of the scalar product between each spectrum, $\mathbf{q}_k^{(0)}$, in $\mathbf{Q}^{(0)}$ and all possible shifts of the eigenvector, $\mathbf{J}_\theta\tilde{\phi}^{(i)}$, with respect to θ is done using a grid search of θ in the interval $[-\Theta, \Theta]$ in order to find the frequency shift $\hat{\theta}_k^{(i)}$

$$\hat{\theta}_k^{(i)} = \arg\max_\theta |(\mathbf{J}_\theta\tilde{\phi}^{(i)})^{\mathbf{H}}\mathbf{q}_k^{(0)}| \quad (8)$$

The corresponding amplitude scaling function $\hat{a}_k^{(i)}$ is then the inner product

$$\hat{a}_k^{(i)} = (\mathbf{J}_{\hat{\theta}_k^{(i)}}\tilde{\phi}^{(i)})^{\mathbf{H}}\mathbf{q}_k^{(0)} \quad (9)$$

The vector $\mathbf{q}_k^{(0)}$ is prolonged with Θ extra samples in both the beginning and the end in the same way as for the eigenvector above and each shifted spectrum, $\mathbf{q}_k^{(i+1)}$, in $\mathbf{Q}^{(i+1)}$ is constructed as

$$\mathbf{q}_k^{(i+1)} = \mathbf{J}_{-\hat{\theta}_k^{(i)}}\mathbf{q}_k^{(0)} \quad (10)$$

The purpose of this shift is to generate a TFD with a higher energy concentration around the dominant frequency in order to calculate an even more concentrated spectral profile (principal eigenvector of the correlation matrix).

The total energy of the amplitude scaling factors is used to detect convergence of the procedure. When the difference in total energy between the present and previous iteration is less than 10% of the total energy for the previous iteration, the iterations are terminated. Otherwise $i = i+1$ and the procedure is repeated from eqn. (6).

At the final iteration, a *homogeneity function*, $\hat{\delta}_k$, is calculated in order to measure how well the spectral profile represents each spectra in the time-frequency distribution

$$\hat{\delta}_k = \frac{(\mathbf{J}_{\hat{\theta}_k^{(i)}} \tilde{\phi}^{(i)})^{\mathbf{H}} \mathbf{q}_k^{(0)}}{\| (\mathbf{J}_{\hat{\theta}_k^{(i)}} \tilde{\phi}^{(i)}) \| \| \mathbf{q}_k^{(0)} \|} \tag{11}$$

This measure is calculated both for the entire frequency interval (2.5-25 Hz) representing mainly the fundamental frequency ($\hat{\delta}_{1,k}$) but also for the frequency interval 9-25 Hz which contains the harmonics in order to measure how well the shape matches the shape represented by the spectral profile ($\hat{\delta}_{2,k}$).

D. Postprocessing

One example of postprocessing is to investigate modulatory properties in the atrial signals. This can be done by performing spectral analysis of the frequency trend, $\hat{s}_k$, i.e. the frequency of the fundamental peak in the spectral profile, $\phi = [\phi_0 \dots \phi_{N_f-1}]$, adjusted by the frequency shift function

$$\hat{s}_k = f_{\arg\max_i \phi_i - \hat{\theta}_k} \tag{12}$$

The "modulation" spectrum $\mathbf{S}$ of $\hat{s}_k$ is calculated using the FFT.

III. Results

In order to investigate the properties of different atrial arrhythmias, the proposed method was applied in 20 recordings from patients with atrial arrhythmias. Three patients had atrial tachycardia and 17 had chronic atrial fibrillation. Three patients were studied during rhythm controlled respiration (0.125 Hz) for the purpose of investigating possible modulatory effects on the fibrillation frequency. Each recording was of one minute duration and lead V_1 was investigated (although V_2 and V_3 were also used for QRST cancellation).

The performance of the proposed method is demonstrated by four examples which are representative for the entire database. Five seconds from each of the four ECG signals in which the ventricular activity has been cancelled are shown in Fig. 1. The corresponding decomposed time-frequency distributions are presented in Figs. 2-5. Each of these figures shows the logarithmic STFT in the upper left corner. The resulting spectral profile, ϕ, is plotted below the STFT; for comparison the power spectrum is also presented. In the upper right corner the three trends $\hat{\theta}_k$, $\hat{a}_k$ and $\hat{\delta}_k$ are plotted. In the rightmost plot the two homogeneity functions $\hat{\delta}_{1,k}$ and $\hat{\delta}_{2,k}$ are plotted. Finally, in the lower right corner the spectrum, $\mathbf{S}$, of the frequency trend, $\hat{s}_k$, is shown and is of particular interest in examples 3 and 4.

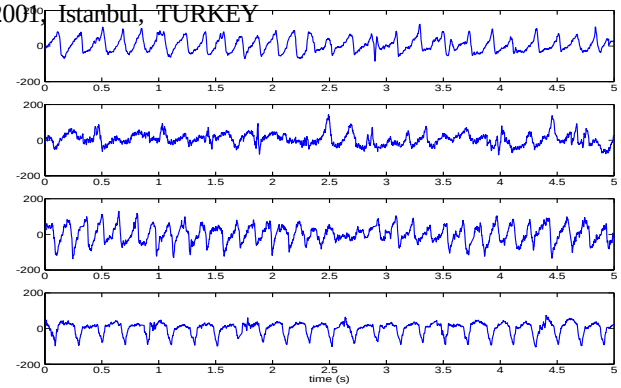

Fig. 1. Five seconds from each of the four ECG signals in which the ventricular activity has been cancelled.

A. Example 1

Figure 2 shows a typical case of atrial fibrillation with a dominant fundamental frequency at around 6 Hz. The frequency shift function reveals a relatively large frequency variation. In this case, two harmonics are visible in the spectral profile. From the homogeneity functions in the rightmost plot it is noted that the spectral profile relatively well represents each spectra in the distribution both concerning the fundamental and the harmonic pattern (the average values of $\hat{\delta}_{1,k}$ and $\hat{\delta}_{2,k}$ are 0.94 and 0.90 respectively). This means that the shape of the fibrillation waves are relatively constant independently of frequency and amplitude. In this example no obvious patterns can be seen in the amplitude function and the frequency trend spectrum. Comparing the power spectrum of the entire one-minute signal to the spectral profile, it is evident that the fundamental peak of the latter spectrum is narrower and that the harmonics can be more easily discerned.

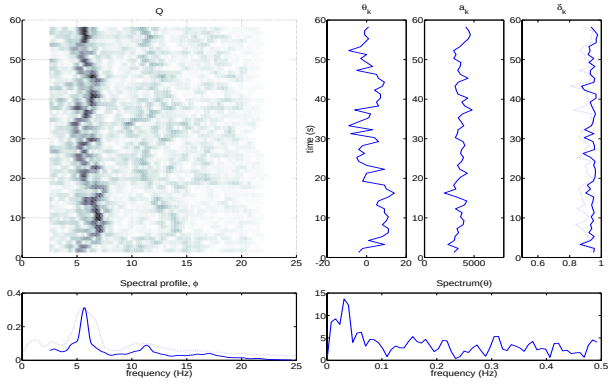

Fig. 2. Example 1: Decomposition of a 1 minute atrial fibrillation time-frequency representation. Upper right: frequency shift function, amplitude function and homogeneity function (2.5-25 Hz - solid line, 9-25 Hz - dotted line). Lower left: Spectral profile - solid line, power spectrum - dotted line. Lower right: Power spectrum of the frequency trend s_k.

B. Example 2

The second example is shown in Fig. 3. In this case the typical irregular rhythm of atrial fibrillation was suddenly

interrupted by 10 seconds of a more regular rhythm. This can be easily detected in both the constant value of the frequency shift trend but also in the sudden increase of the amplitude trend. Similar to example 1, two harmonics are observed but the peaks are here more distinct. Again, a large difference is found between the power spectrum and the spectral profile. It is interesting to note that both homogeneity measures are slightly closer to one during the regular rhythm. In average the homogeneity values were $\hat{\delta}_1 = 0.92$ and $\hat{\delta}_2 = 0.88$ indicating that the shape given by the spectral profile also is representative during weaker irregular rhythm. No obvious peak is detected in the frequency trend spectrum. However, it is curious to note that the largest peak above 0.1 Hz occur at the normal respiratory rate (0.31 Hz and 0.32 Hz respectively) in Figs. 2 and 3.

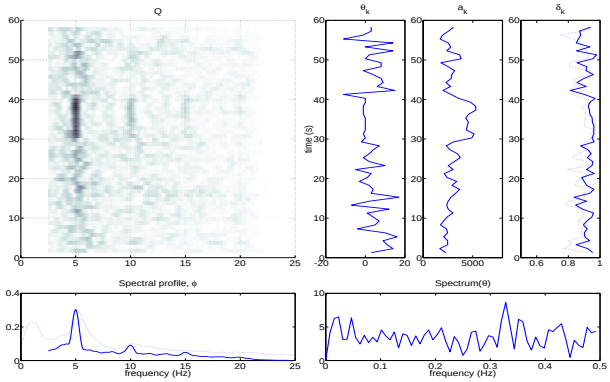

Fig. 3. Example 2: Decomposition of a 1 minute atrial fibrillation time-frequency representation with a 10 s long regular rhythm at 30 s. (See legend to Figure 1 for further explanations.)

C. Examples 3 and 4

Figure 4 shows a decomposed atrial fibrillation signal with two harmonics in the spectral profile and with relatively large frequency shifts (θ_k). In Fig. 5, the same decomposition is performed for an atrial tachycardia case (3 harmonics combined with an almost constant frequency shift function (θ_k)). A number of observations can be made: First, the homogeneity measures are as expected in average very close to one ($\hat{\delta}_1 = 0.99$ and $\hat{\delta}_2 = 0.97$) for the more regular arrhythmia in Fig. 5, but also for the atrial fibrillation case in Fig. 4 these measures are relatively high ($\hat{\delta}_1 = 0.95$ and $\hat{\delta}_2 = 0.91$). Secondly, in both cases obvious peaks are present around the respiratory frequency at 0.125 Hz in the frequency trend spectra. This peaks disappeared when blocking the autonomous nervous system with atropine. Finally, when the rhythm is stationary the power spectrum and the spectral profile are identical.

IV. Conclusions

Analysis of signals from a database of 20 patients with different arrhythmias (mainly atrial fibrillation) shows that the proposed method is very well-suited for characterizing the atrial signals as indicated by the homogeneity trends.

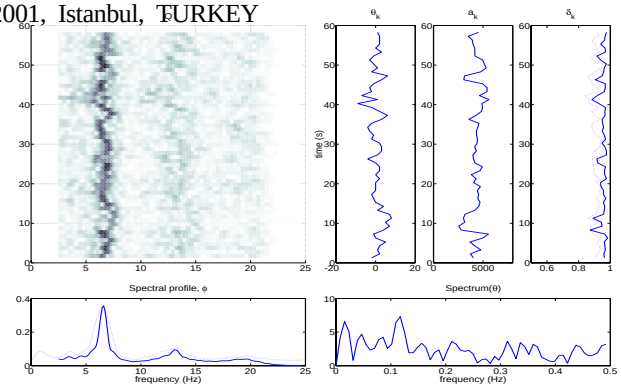

Fig. 4. Example 3: Decomposition of a 1 minute atrial fibrillation time-frequency representation during rhythm control respiration (0.125 Hz). (See legend to Figure 1 for further explanations.)

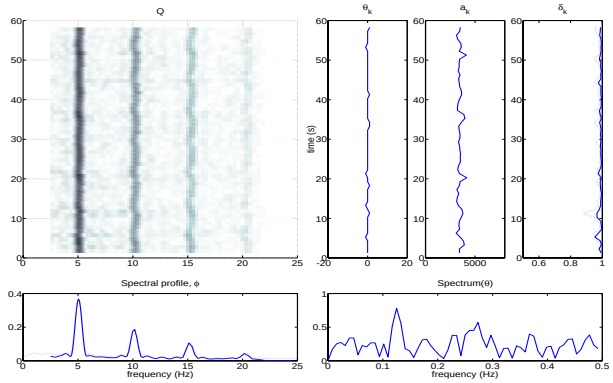

Fig. 5. Example 4: Decomposition of a 1 minute atrial tachycardia time-frequency representation during rhythm control respiration (0.125 Hz). (See legend to Figure 1 for further explanations.)

Comparing the power spectrum of the signals to the spectral profiles, it is evident that the peaks of the spectral profiles are narrower and that the peaks of the harmonics, representing atrial morphology, can be much better discerned. Further, modulatory properties in the atrial activity can be obtained from the frequency trend.

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